

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/22

Paper 2, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	(i)	$f(-2) = -32 - 16 + 30 + 18 = 0$	B1	All four evaluated terms must be seen. Allow if correct long division used
	(ii)	$f(x) = (x+2)(4x^2 - 12x + 9)$	M1	Coefficients 4 and 9
		$= (x+2)(2x-3)(2x-3)$	A1	Coefficient -12
		$f(x) = 0 \rightarrow x = -2, 1.5$ nfww	A1	All three factors together
			A1	Allow 1.5 mentioned just once
2	(i)	$(2-3x)^6 = 64 - 576x + 2160x^2$ isw	B1B1B1	
	(ii)	$2160 - 2 \times 576 = 1008$	M1 A1	<i>their</i> final $2160 + 2 \times$ <i>their</i> final -576
3	(i)	$\overrightarrow{AB} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$	B1	Allow \overline{BA} May be implied by later work.
		$ AB = \sqrt{15^2 + 8^2} (=17)$	M1	Use of Pythagoras on <i>their</i> AB
		Speed = $17 \times 3 = 51$ km/hr	A1	Must be exact
	(ii)	$\overrightarrow{BC} = \begin{pmatrix} 16 \\ -30 \end{pmatrix}$	B1	Allow \overline{CB}
		$ BC = \sqrt{16^2 + 30^2} (=34)$	M1	Use of Pythagoras on <i>their</i> BC
		Time taken = $\frac{34}{51} \times 60 = 40$ mins (or $\frac{2}{3}$ hrs)	A1	Allow answers which round to 40 to 2sf. Accept 0.66 or 0.67 hrs. Mark final answer.

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<p>4 (a)</p> <p>(b) (i)</p> <p>(ii)</p>	$2\mathbf{BA} = 2 \begin{pmatrix} 1 & -2 & 4 \\ -2 & 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 5 \\ 7 & 4 \end{pmatrix}$ $= 2 \begin{pmatrix} 24 & 5 \\ 5 & 17 \end{pmatrix} = \begin{pmatrix} 48 & 10 \\ 10 & 34 \end{pmatrix}$ $\mathbf{C}^{-1} = \frac{1}{8} \begin{pmatrix} 6 & -2 \\ 1 & 1 \end{pmatrix} \text{ isw}$ $\mathbf{I} - \mathbf{D} = \begin{pmatrix} -2 & 2 \\ -1 & -3 \end{pmatrix}$ $\mathbf{X} = \mathbf{C}^{-1}(\mathbf{I} - \mathbf{D}) = \frac{1}{8} \begin{pmatrix} 6 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -2 & 2 \\ -1 & -3 \end{pmatrix}$ $= \frac{1}{8} \begin{pmatrix} -10 & 18 \\ -3 & -1 \end{pmatrix} \text{ isw}$	<p>B3,2,1,0</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>-1 each error in 2×2 result. Failure to multiply by 2 is one error</p> <p>$\frac{1}{8}$</p> <p>Matrix</p> <p>Pre multiply <i>their</i> $\mathbf{I} - \mathbf{D}$ with <i>their</i> \mathbf{C}^{-1}</p>
<p>5 (a)</p> <p>(b)</p>	$2^{3(q-1)} \times 2^{2p+1} = 2^{14}$ $3^{2(p-4)} \times 3^q = 3^4$ <p>Solve $3q + 2p = 16$ $q + 2p = 12$</p> $p = 5, \quad q = 2$ $(3x - 2)(x + 1)$ $= 50$ $3x^2 + x - 52 = 0 \rightarrow (3x + 13)(x - 4)$ $x = 4$ $x = -\frac{13}{3} \text{ discarded}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct powers of 2 allow unsimplified isw</p> <p>Correct powers of 3 allow unsimplified isw</p> <p>Attempt to solve <i>their</i> linear equations by eliminating one variable</p> <p>Both correct</p> <p>LHS oe isw</p> <p>50 from correct processing of $2 - \lg 2$</p> <p>Solution of <i>their</i> three term quadratic</p> <p>Roots must be obtained from correct quadratic</p>

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6	(i)	$a = 3, b = 2, c = 4$	B1B1B1	
	(ii)	$\frac{dy}{dx} = 8 \cos 4x$ isw	M1 A1FT	$\pm k \cos cx$ and no other term in x $c \neq 1$ $bc \times \cos cx$ and no other term
	(iii)	$x = \frac{\pi}{2} \rightarrow \frac{dy}{dx} = 8 \cos 2\pi = 8$	DM1	Find <i>their</i> correct numerical $\frac{dy}{dx}$
		Eqn: $\frac{y-3}{x-\frac{\pi}{2}} = -\frac{1}{8} \quad \left(\rightarrow y = -\frac{1}{8}x + 3.20 \right)$	M1 A1	Find equation with <i>their</i> numerical normal gradient ie $-\frac{1}{8}$ and point $\left(\frac{\pi}{2}, 3 \right)$ All correct isw
7	(i)	$\frac{h}{8} = \frac{6-r}{6} \rightarrow h = \frac{4}{3}(6-r)$	M1 A1	Uses correct ratio. Cannot be implied
	(ii)	$V = \pi r^2 h = \pi r^2 \times \frac{4}{3}(6-r)$ $= 8\pi r^2 - \frac{4}{3}\pi r^3$	B1	AG all steps must be seen Penalise missing brackets at any point in working
	(iii)	$\frac{dV}{dr} = 16\pi r - 4\pi r^2$	M1 A1	Differentiate at least one power reduced by one
		$\frac{dV}{dr} = 0 \rightarrow r = 4$ $V = \frac{128}{3}\pi \quad (= 42.7\pi)$ $\frac{d^2V}{dr^2} = 16\pi - 8\pi r < 0$ when $r = 4 \rightarrow \max$	M1 A1 A1 B1	Attempt to solve – must get $r = \dots$ Correct value of r . Ignore $r = 0$ Correct value of V . Condone 134. $\frac{d^2V}{dr^2}$ must be correct and some indication of a negative value seen plus maximum stated

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8	(i)	Gradient $AB = \frac{8-2}{9+3} \quad \left(= \frac{1}{2} \right)$ isw Equation AB and $x=0 \rightarrow \frac{y-2}{0+3} = \frac{1}{2} \quad \left(\rightarrow y = \frac{1}{2}x + 3.5 \right)$ $\rightarrow y = 3.5$	B1	
			M1	
			A1	
	(ii)	D is $(3, 5)$	B1	
	(iii)	Gradient perpendicular $= -2$ Equation perpendicular $\frac{y-5}{x-3} = -2$ $\rightarrow (y = -2x + 11)$	M1	
		A1		
	(iv)	E is $(0, 11)$	A1FT	
	(v)	Area of $ABE = \frac{1}{2} \begin{vmatrix} -3 & 9 & 0 & -3 \\ 2 & 8 & 11 & 2 \end{vmatrix}$ $= \frac{1}{2} \begin{vmatrix} -24 + 99 - 18 + 33 \end{vmatrix} = 45$ Area of $EDC = \frac{1}{2} \begin{vmatrix} 3 & 0 & 0 & 3 \\ 5 & 3.5 & 11 & 5 \end{vmatrix}$ $= \frac{1}{2} \begin{vmatrix} -10.5 + 33 \end{vmatrix} = 11.25$	M1	For area of ABE or ECD . $\frac{1}{2}$ and <i>their</i> correct 8 elements must be seen.
			A1	45 condone from $E(0, -4)$
			A1	11.25 condone from $E(0, -4)$

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<p>9 (i)</p> <p>(ii)</p> <p>(iii)</p>	$\tan 2x = -\frac{5}{4}$ $(2x = 128.7, 308.7)$ $x = 64.3 \text{ awrt}$ 154.3 awrt $\operatorname{cosec}^2 y + 3\operatorname{cosec} y - 4 = 0 \quad \text{or}$ $4\sin^2 y - 3\sin y - 1 = 0$ $(\operatorname{cosec} y + 4)(\operatorname{cosec} y - 1) = 0 \quad \text{or}$ $(4\sin y + 1)(\sin y - 1) = 0$ $\sin y = -\frac{1}{4} \quad \text{or} \quad \sin y = 1$ $y = 194.5, 345.5, 90$ $z + \frac{\pi}{4} = \pi - \frac{\pi}{3} \quad \text{or}$ $z + \frac{\pi}{4} = \pi + \frac{\pi}{3}$ $z = \frac{5\pi}{12}, \frac{13\pi}{12}$	<p>M1</p> <p>A1 A1FT</p> <p>B1</p> <p>M1</p> <p>A1A1A1</p> <p>B1</p> <p>B1</p> <p>B1B1</p>	<p>For obtaining and using $\tan 2x = \pm \frac{5}{4}$ or $\pm \frac{4}{5}$ resulting in $2x =$ $\tan x = \dots$ gets M0 <i>their</i> $64.3^\circ + 90^\circ$</p> <p>In any form as a three term quadratic.</p> <p>Solve three term quadratic in $\operatorname{cosec} y$ or $\sin y$ Answers must be obtained from the correct quadratic</p> <p>Accept 2.09, 2.10, $\pi - 1.05$, $\pi - 1.04$ on RHS. Could be implied by final answer Accept 4.19, 4.18, $\pi + 1.05$, $\pi + 1.04$ on RHS. Could be implied by final answer Answers must be correct multiples of π.</p>
<p>10 (i)</p> <p>(ii)</p> <p>(iii)</p>	$s = \frac{1}{2}e^{2t} + 3e^{-2t} - t + (c)$ $t = 0, s = 0 \rightarrow c = -3.5$ $\left(s = \frac{1}{2}e^{2t} + 3e^{-2t} - t - 3.5 \right)$ $v = 0 \rightarrow u^2 - u - 6 = 0 \quad \text{oe}$ $(u - 3)(u + 2) = 0$ $\rightarrow u = 3 \rightarrow t = \frac{1}{2} \ln 3 \quad \text{or} \quad 0.549$ $t = \frac{1}{2} \ln 3 \rightarrow a = 2e^{2t} + 12e^{-2t}$ $= 6 + 4 = 10$	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>Integrate : coefficient of $\frac{1}{2}$ or 3 seen with no change in powers of e. Ignore $-t$</p> <p>All correct and simplified</p> <p>Obtain three term quadratic in u or e^{2t} Condone sign errors.</p> <p>Solve three term quadratic</p> <p>Accept 0.55 No second answer</p> <p>Correct differentiation</p> <p>Allow awrt 10.0 or 9.99. No second answer.</p>